3. Fluid Dynamics

The equations of motion are best written in the form of integrals through the depth of the curved jet. The variable of integration is the normal coordinate n, perpendicular to the streamwise coordinate s as shown in Fig. 4. The density of the water is ρ, constant throughout the flow under conditions attainable by Olsen and Thomas's rock cutter. The speed of the water is u and the pressure is p at the location (s,n). Along the interface (s,d) between air and water, p must equal the atmospheric pressure p_a. Streamline curvature raises p to some higher value p_s at the cutting surface (s,0). The water speed u is uniform across the jet at s = 0 and has a value u_o related to the stagnation pressure P_o in the pressure intensifier by the Bernoulli equation:

$$P_o - p_a = \frac{1}{2} \rho u_o^2 . \tag{6}$$

The width of the jet is constant by assumption, so the equation of volume conservation takes the form

$$\int_0^d u \, dn = u_o d_o . \tag{7}$$

Equation (7) could be used to calculate the local depth d of the stream, but (7) is not needed to determine h in the present theory and will not be seen again.

Conservation of momentum normal to the streamlines results in a pressure balance:

$$p_s - p_a = \frac{1}{R} \int_0^d \rho u^2 \, dn . \tag{8}$$

It is at this stage that approximation (5) first enters the analysis. If the jet were not thin compared with its radius of curvature, then the variation of R from one streamline to the next would have to be taken into account, and R would have to be included under the integral in equation (8). Under approximation (5), all streamlines share a common radius of curvature R at station s.

It is worthwhile to examine the magnitude of the hydrodynamic pressure p_s against the cutting surface. Suppose P_o is 1000 atm, that is $P_o = 1000 \; p_a$. The momentum flux ρu^2 is of order ρu_o^2, which is about 2 P_o according to (6). Thus $\rho u^2 \approx 2000 \; p_a$. The ratio d/R is assumed small, say d/R = 0.1, so $p_s \approx 200 \; p_a$.